D. Applications of Tunneling

(a) α-decay in Nuclei

Data of 5 α-particle emitting nuclei

Nucleus	k.e. (MeV)	half-life $t_{1/2}$
^{216}Ra	9.5	0.18 μs
^{194}Po	7.0	0.7 s
^{240}Cm	6.4	27 days
^{226}Ra	4.9	1600 years
^{232}Th	4.1	14 billion years

α-particle k.e. don't vary much!

$t_{1/2}$ vary by a lot!

↕ Many orders of magnitude

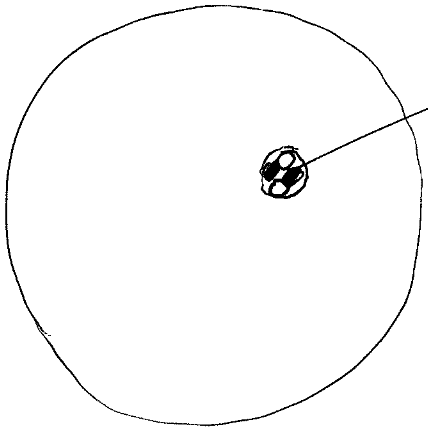
Note correlation:

longer $t_{1/2}$ ↔ smaller α-particle k.e.

shorter $t_{1/2}$ ↔ higher α-particle k.e.

Back to tunneling and α-decay

- Gamov (1928) : α-decay comes from tunneling



$^{4}_{2}He$ "grouped" occasionally

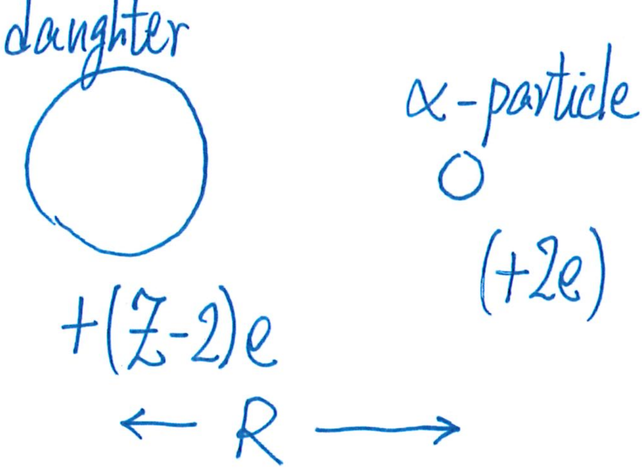
- travels toward surface

  ⇒ an attempt of tunneling

  [e.g $^{232}_{92}U \rightarrow ^{228}_{90}Th + \alpha$]

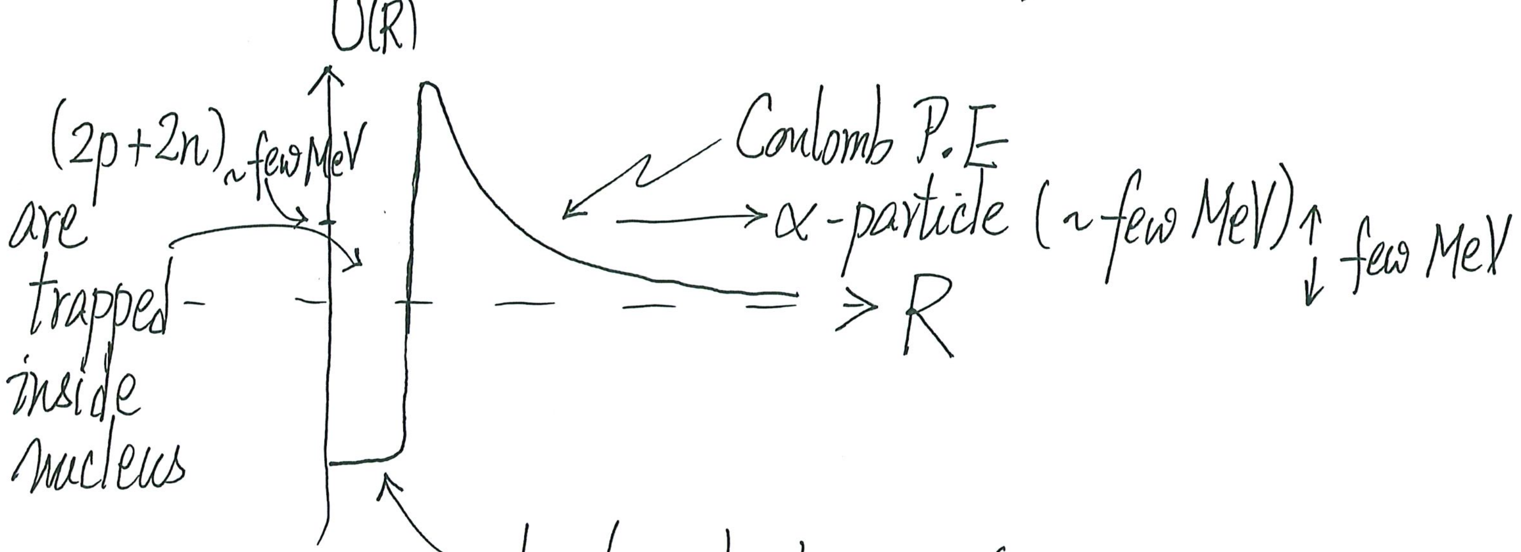
- But other nucleons ($^{228}_{90}Th$) exert attraction (nuclear force) and don't want α to leave ⇒ a barrier
- Once outside : α (+ve charge) and daughter nucleus (+ve charge) repel ⇒ $^{228}_{90}Th$ α

  don't want α to come back



Coulomb repulsion

$$\left[P.E. = + \frac{(Z-2)\cdot 2e^2}{4\pi\varepsilon_0 R} \right]$$

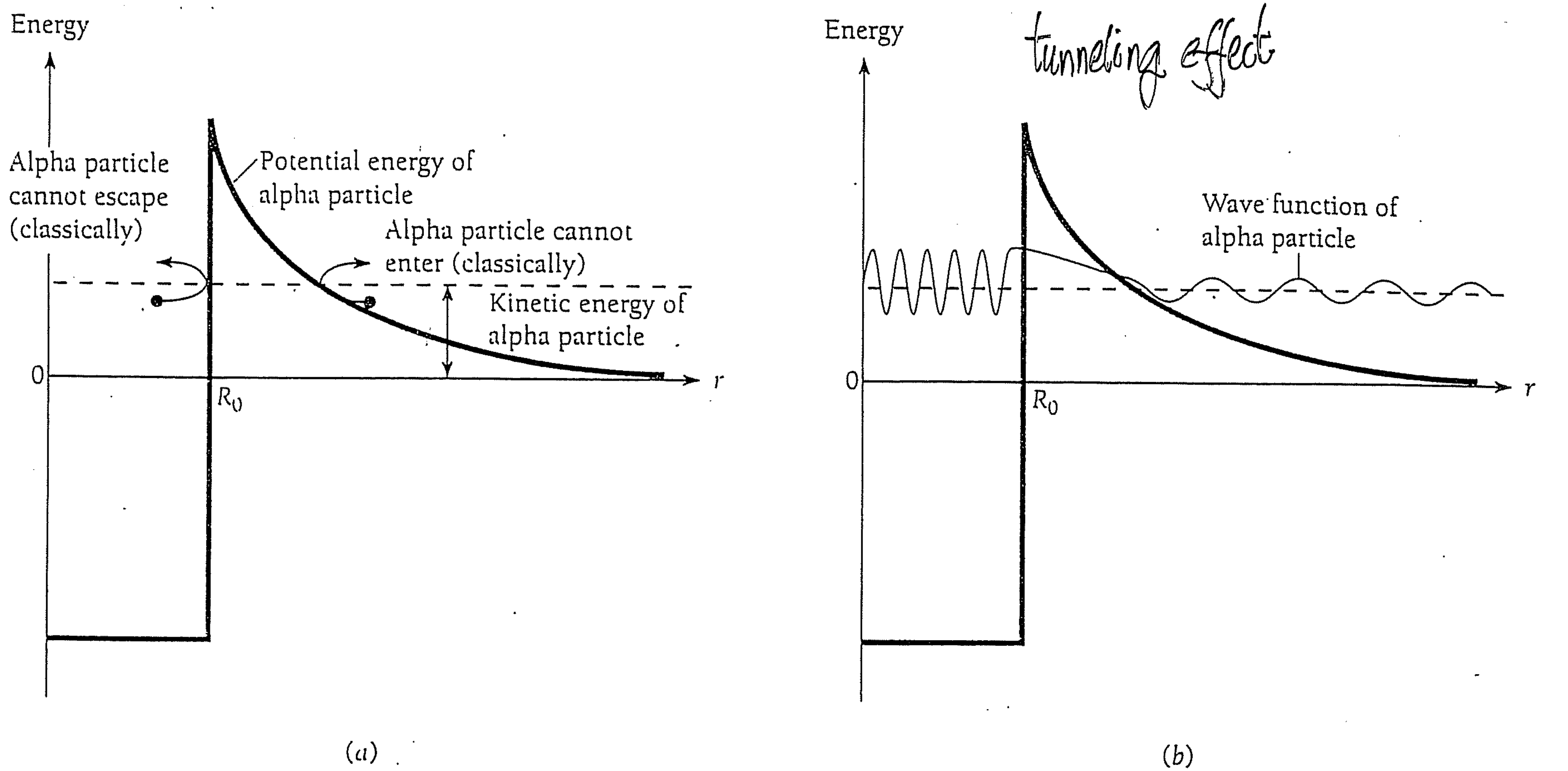


due to short-range ($\sim 10^{-15}$ m) nuclear force

Q: Does α-particle climb out of the barrier?

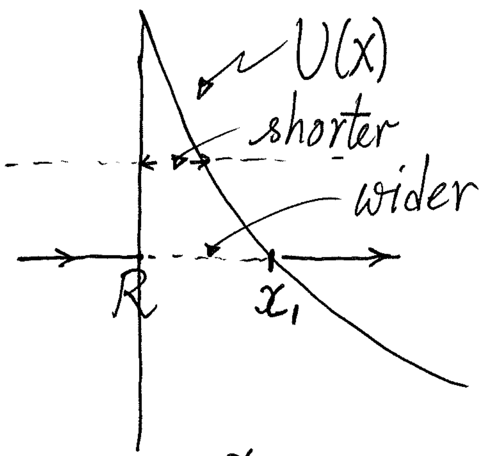
[No! α-particle k.e. is NOT that high!]

Q: How does α-decay occur? [Tunneling]

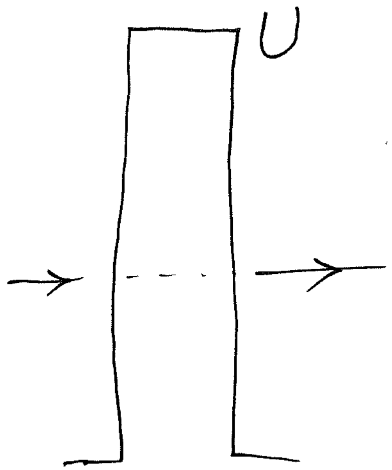


(*a*) In classical physics, an alpha particle whose kinetic energy is less than the height of the potential barrier around a nucleus cannot enter or leave the nucleus, whose radius is R_0. (*b*) In quantum physics, such an alpha particle can tunnel through the potential barrier with a probability that decreases with the height and thickness of the barrier.

This is why α-particles (a few MeV) can be detected even the barrier is higher.



Not



$$\sim e^{-2\int_R^{x_1}\sqrt{\frac{2m}{\hbar^2}(U(x)-E)}\,dx} \qquad\qquad \sim e^{-2L\sqrt{\frac{2m}{\hbar^2}(U-E)}}$$

[slight increase in E changes the value of T by orders of magnitude!]

<u>E.g.</u> $Z = 90$, $E \sim 6\,\text{MeV}$, $R \sim 8\,\text{fm}$

$T \sim 10^{-29}$ (tiny!)

↑ prob. of getting through per attempt (interpretation)

- So $T \sim e^{-2L\sqrt{\frac{2m}{\hbar^2}(U-E)}}$ is tiny. Why could we see α-decays?

  ↳ prob. of getting through per attempt

- Although T is tiny, α-particle keeps on trying!

  Estimate: Diameter of nucleus $= D \sim 15$ fm (big nucleus)

  α-particle's speed inside nucleus $\sim c/10$ (roughly)

  f = # attempts per second $\sim \dfrac{c/10}{15\text{ fm}} \sim 10^{21}\ s^{-1}$ — try 10^{21} times per second

  $\therefore \lambda$ = Prob. of a nucleus to decay per second

  λ (s^{-1}) $= \dfrac{1}{\tau} = T \cdot f$ $\qquad (\tau \cdot \ln 2 = t_{1/2})$

  T: tiny (tunneling); f: big number (s^{-1})